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EFFECT OF STRUCTURAL AND MECHANICAL CHARACTERISTICS OF THE COMPOSITE MATERIAL ON THE DEFORMATION OF A REFLECTOR ANTENNA

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This is a study of the effect of structural and mechanical characteristics of a composite material on the stress–strain state of a reflector antenna shaped as a composite thin shell of revolution subjected to gravity, wind, and temperature loads. The boundary-value problem for the system of partial differential equations governing the behavior of this structure is reduced to a sequence of boundary-value problems for inhomogeneous systems of ordinary differential equations with variable coefficients. The resulting stiff systems of equations are solved by Godunov's method of discrete orthogonalization.

Formulation of the Problem the and Method of the Solution. The reflector antenna is an important element of satellite systems, which are widely used to ensure effective cordless communication. The main requirements imposed on these antennas are strength and minimum deviation of the reflector profile from a specified shape. The use of composite materials (CM) opens up wide opportunities for meeting these requirements.

We consider a reflector antenna shaped as a reinforced thin parabolic shell of revolution with a focal distance f, aperture diameter D, and thickness $2h$. We study the effect of structural and mechanical characteristics of the composite on the behavior of the structure under the same loading and fixing conditions and geometry and linear dimensions of the shell.

To model the reflector, we use the structural model of a reinforced layer, structural failure criterion [1], and classical linear model of a thin shell. The boundary-value problem is formulated for a system of 19 algebraic and partial differential equations for 19 unknown functions. The initial boundary-value problem is reduced to a sequence of boundary-value problems for systems of ordinary differential equations by the method of separation of variables using the trigonometric basis [2]. Each system of ordinary differential equations can be written in the general form

$$\frac{d\boldsymbol{y}_m}{dr} = A_m(r)\boldsymbol{y}_m + \boldsymbol{b}_m(r), \tag{1}$$

where $\boldsymbol{y}_m(r)$ is the vector function of the expansion coefficients for the mth harmonic, r is the distance from the reference surface to the axis of revolution, $A_m(r)$ is the 8×8 matrix of the system, and $\boldsymbol{b}_m(r)$ is the free-term vector. System (1) and the boundary conditions

$$G_l\boldsymbol{y}_m(r_{\min}) = \boldsymbol{g}_{l,m}, \qquad G_r\boldsymbol{y}_m(r_{\max}) = \boldsymbol{g}_{r,m} \tag{2}$$

(G_l and G_r are 8×4 matrices and $\boldsymbol{g}_{l,m}$ and $\boldsymbol{g}_{r,m}$ are vectors of dimension 4) constitute a closed boundary-value problem.

Systems (1) are stiff systems by virtue of their high order and the presence of such small parameters as h/R_i and E_0/E_n (R_i are the principal radii of curvature and E_0 and E_n are Young's moduli of the binding and reinforcing fibers of the nth family, respectively). Moreover, for shells of nonzero Gaussian curvature with reinforcement parameters variable in the meridional direction, the matrix of the system strongly depends on the meridional coordinate. This implies that the ratio $\Lambda_m(r) = \max\limits_j |\lambda_{j,m}(r)|/\min\limits_j |\lambda_{j,m}(r)|$, where $\lambda_{j,m}(r)$ are the eigenvalues of the matrix $A_m(r)$, is much greater than unity.

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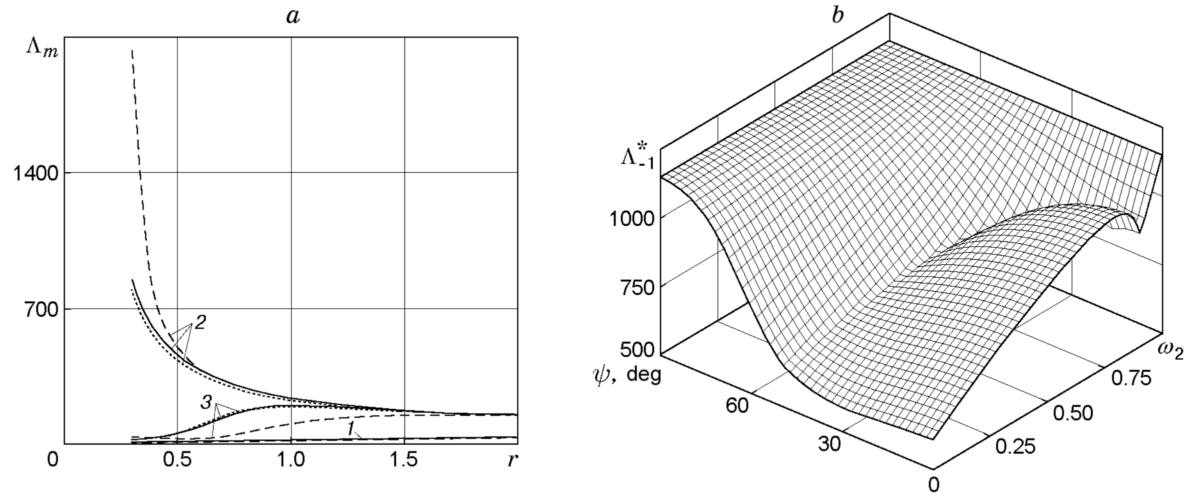


Fig. 1

Figure 1a shows the quantities Λ_m as functions of the radius and harmonic number for the calculated stress–strain state of aluminum–carbon (solid curves), titanium–carbon (dotted curves), and carbon–plastic (dashed curves) reflectors. Curves 1 (which coincide for three types of reflectors) refer to $m = 0$, curves 2 to $m = -1$ and 1, and curves 3 to $m = -2$ and 2. The quantity $\Lambda_m(r)$ strongly depends on the meridional coordinate and mechanical characteristics of the CM. Moreover, this dependence changes both quantitatively and qualitatively for different harmonics. The maximum values correspond to antisymmetric components ($m = -1$ and 1). Figure 1b shows the dependence of the quantity $\Lambda^*_{-1} = \max_r\{\Lambda_{-1}(r)\}$ on structural parameters of the CM, calculated for antisymmetric components of the stress–strain state of the titanium–carbon reflector. This dependence demonstrates a considerable effect of the reinforcement structure on the stiffness of the system, which changes twofold in Fig. 1b. Here ω_2 is the specific intensity of circumferential fibers and $\psi_1 = -\psi_3 = \psi$ are the reinforcing angles of spiral fibers.

The boundary-value problem for a stiff system of differential equations is solved by the method of discrete orthogonalization proposed by Godunov [3].

Stress–Strain State of the Reflector Under Gravity. We consider a parabolic reflector with an aperture diameter of 4 m and a focal distance of 1.5 m, which is rigidly fixed at the center over the radius of 0.3 m and loaded by gravity. The antenna axis is tilted at an angle $\beta = 30°$ to the Earth's surface. The structure is made of aluminum (density is $\rho_0 = 2.68 \cdot 10^3$ kg/m^3, $E_0 = 70$ GPa, and ultimate strength is $\sigma^*_0 = 170$ MPa) reinforced by high-modulus carbon fibers ($\rho_n = 1.9 \cdot 10^3$ kg/m^3, $E_n = 780$ GPa, and $\sigma^*_n = 2.5$ GPa) with a volume fraction $\omega_a = 0.3$. The reinforcement consists of one circumferential and two spiral families. The specific intensities of reinforcing of the spiral families ω_1 and ω_3 are related to ω_2 by the formula $\omega_1 = \omega_3 = (1 - \omega_2)/2$. The density averaged over the thickness is $\rho \approx 2.446 \cdot 10^3$ kg/m^3 and the reflector thickness is $2h = 15 \cdot 10^{-3}$ m. The gravity is decomposed into axisymmetric and antisymmetric loads governed by systems (1), (2) for $m = 0$ and $m = -1$, respectively. The reduced loads are given by

$$q_{1,0} = -2h\rho g \sin\beta \sin\theta, \qquad q_{2,0} = 0, \qquad q_{3,0} = 2h\rho g \sin\beta \cos\theta,$$

$$q_{1,-1} = -2h\rho g \cos\beta \cos\theta, \qquad q_{2,-1} = -2h\rho g \cos\beta, \qquad q_{3,-1} = -2h\rho g \cos\beta \sin\theta.$$

Here q_1, q_2, and q_3 are the meridional, circumferential, and normal components, respectively, and θ is the angle between the normal to the shell surface and the axis of revolution.

Figure 2a shows the maximum stress intensities in the matrix (bs_0) and spiral (bs_1) and circumferential (bs_2) reinforcement families versus the reinforcing angle of spiral fibers ψ. Figure 2b shows the curves $w_{\max}(\psi)$. The solid curves refer to $\omega_2 = 0$, the dotted curves to $\omega_2 = 0.4$, and the dashed curves to $\omega_2 = 0.8$. The dot-and-dashed curves refer to the maximum deflections and stress intensities in an isotropic aluminum shell. One can see that reinforcement can either improve or deteriorate the rigidity and strength characteristics of the structure. For example, owing to the meridional reinforcement, the stresses in the matrix and deflections decrease by approximately factors of 2 and 1.5, respectively, compared to the aluminum structure.

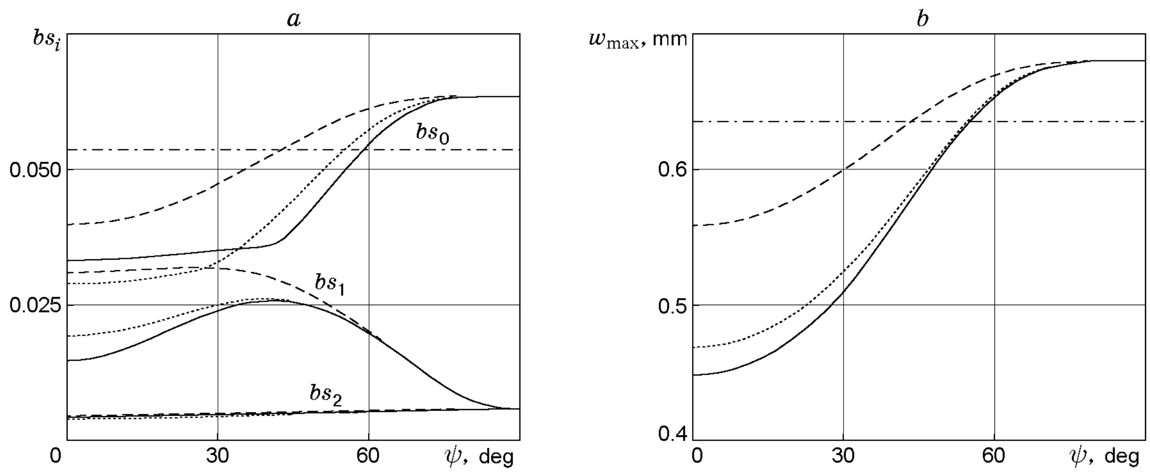


Fig. 2

Loaded by gravity only, the structure remains elastic for all reinforcement parameters. Therefore, in choosing mechanical and structural parameters of CM, one should be guided by the fact that the maximum deflections do not exceed the values established by specifications.

Stress–Strain State of the Reflector Under Gravity, Temperature, and Wind Loads. For reflector antennas, the wind workloads are pressures of 600–800 kg/m^2, which corresponds to wind velocities of the order of 20 m/sec. Moreover, antennas should be designed so as to sustain hurricane winds with velocities of 40 m/sec and higher; in this case, the wind load can reach 3000 kg/m^2 and more.

Like gravity, the wind load can be decomposed into axisymmetric and antisymmetric components. However, since the antisymmetry planes of gravity and wind differ in the general case, the wind load corresponds to harmonics with $m = -1$, 0, and 1. We ignore the load components tangent to the surface. In this case, the components $q_{3,m}$ have the additional terms

$$\Delta q_{3,-1} = v_y p \sin\theta, \qquad \Delta q_{3,0} = -v_z p \cos\theta, \qquad \Delta q_{3,1} = v_x p \sin\theta,$$

where p is the wind load, $\|v_x, v_y, v_z\|$ is the wind direction in the orthonormal coordinate system with the axes Oz (directed along the symmetry axis toward the reflector focus), Ox (parallel to the Earth's surface), and Oy (normal to the Oxz plane and directed away from the Earth's surface).

We consider the case where the reflector of a parabolic antenna with an aperture diameter of 4 m and a focal distance of 1.5 m is shaped as a shell of constant thickness $2h = 15 \cdot 10^{-3}$ m. The antenna axis is tilted at an angle of 30° to the Earth's surface. In addition to gravity, a strong lateral wind acts on the antenna, which corresponds to a pressure $p = 2000$ kg/m^2. Let the reflector be heated to 75°C. We consider the behavior of the aluminum structure (the coefficient of linear expansion is $\alpha_0 = 2.33 \cdot 10^{-5}$ K^{-1}) reinforced by three families of high-modulus carbon fibers ($\alpha_n = 1.5 \cdot 10^{-6}$ K^{-1}).

The wind and temperature loads substantially increase the stresses in the matrix and reinforcement (Fig. 3a). For some structural parameters of the CM, the stress intensity in the aluminum matrix exceeds the critical value. However, this can be avoided by varying the reinforcement parameters. The required rigidity of the mirror can also be ensured by choosing appropriate reinforcement parameters (Fig. 3b).

Let us consider how a similarly reinforced reflector with a titanium matrix ($\rho_0 = 4.5 \cdot 10^3$ kg/m^3, $E_0 =$ 110 GPa, $\alpha_0 = 8.3 \cdot 10^{-6}$ K^{-1}, and $\sigma_0^* = 600$ MPa) behaves under the same conditions. The stresses in the CM elements decrease substantially (Fig. 4a), and owing to the high ultimate strength, the titanium matrix remains elastic for all values of the CM structural parameters. One can see from Fig. 4b that the values of $w_{\max}$ are much lower compared to the aluminum-carbon structure. Nonetheless, the minimum deflections of the reflector with the titanium matrix are 1.5 times higher compared to the aluminum matrix, i.e., the use of a high-modulus matrix does not necessarily increase the rigidity of the structure. We note that, under these loading conditions, plastic strains occur in the isotropic aluminum structure ($bs_0 > 1$), the deflections in the isotropic titanium reflector, which remains elastic, are as large as 5 mm, whereas the deflections of the reflector with an aluminum matrix and carbon fibers attain only 3.3 mm for certain reinforcement parameters.

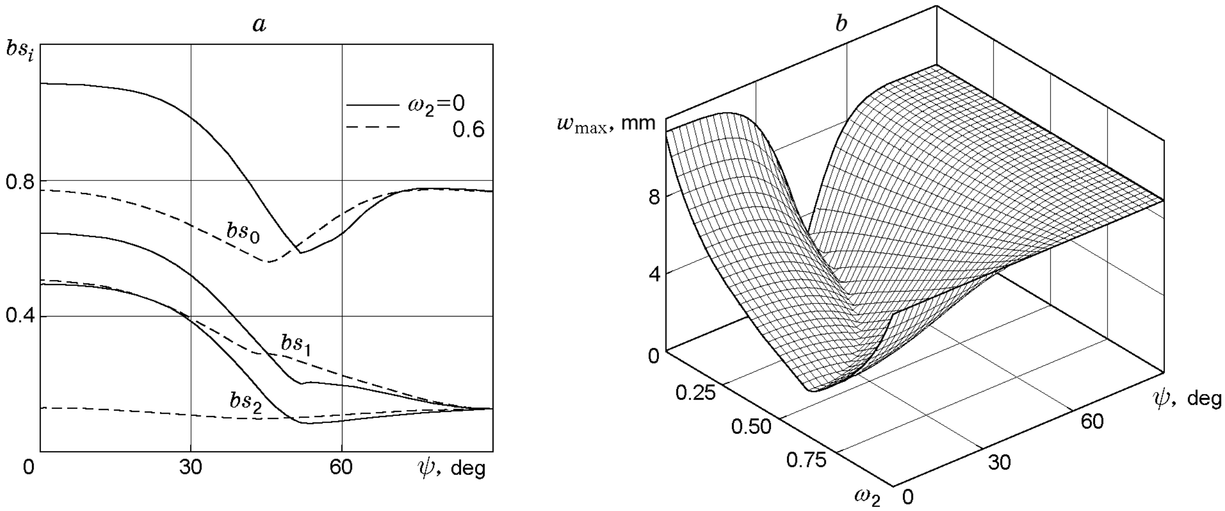
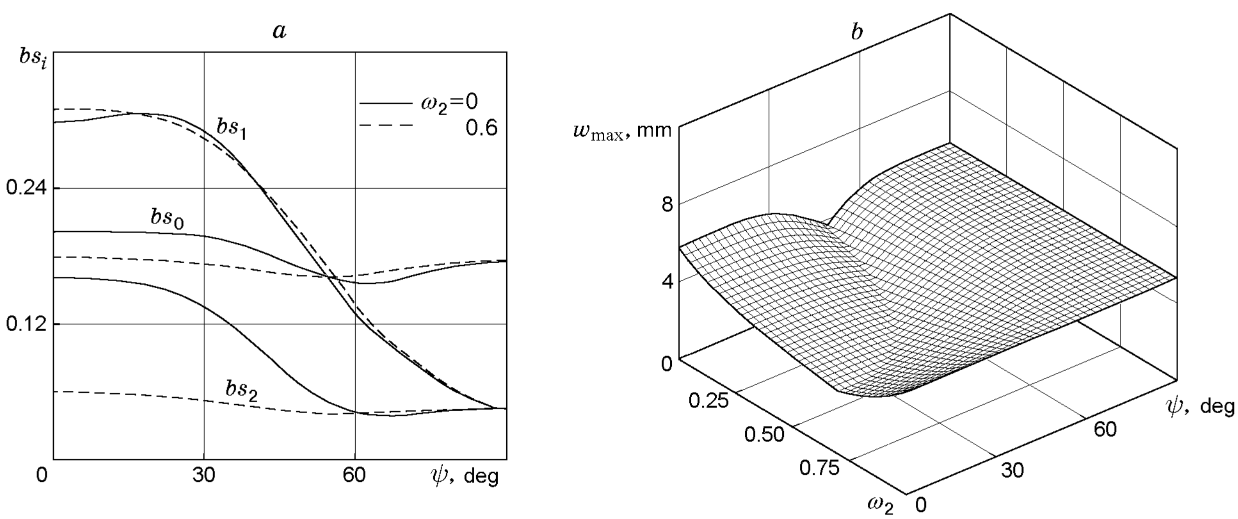

a
b
bs_i
ω_2=0
0.6
0.8
0.4
bs_0
bs_1
bs_2
0
30
60
ψ, deg
$w_{\max}$, mm
8
4
0
0.25
0.50
0.75
ω_2
0
30
60
ψ, deg

Fig. 3


a
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8
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ω_2
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ψ, deg

Fig. 4

Adequacy of Calculations. The question of applicability of the discrete orthogonalization method to solving of the problem of determining the stress–strain state of composite shell structures is of considerable interest. Previously, these problems were solved by the spline-collocation method [4–8] with a known theoretical estimate of the error. Golushko et al. [4, 5] determined the axisymmetric stress–strain state of combined tanks and high-pressure vessels. The calculation results obtained by two methods mentioned above are in excellent agreement.

For the parabolic reflector, the components of the stress–strain state calculated by the discrete-orthogonalization and spline-collocation methods differ by no more than 0.05% (for each harmonic). Decreasing the grid size in the discrete-orthogonalization method and performing the calculations by the spline-collocation method with higher accuracy, we find that the results are closer, the relative differences being as small as 10^{-8}%. The discrete-orthogonalization method is less time consuming by a factor of 3–6 compared to the spline-collocation method.

Conclusions. An analysis of the stress–strain state of the reinforced parabolic reflector loaded by gravity shows that the structure is underloaded. This allows one to choose the reinforcement structure under the rigidity requirements. A comparison with the isotropic aluminum structure shows that the reinforcement can either improve or deteriorate technical performances of the reflector. Wind and temperature loads can cause considerable distortion and even failure of the structure. At the same time, it was found that an appropriate choice of the reinforcement structure can prevent the failure of the reflector and decrease its strains.

It was shown that structural and mechanical characteristics of a CM substantially affect the behavior of the structure: the stresses in the matrix change by approximately a factor of 2, the stresses in the reinforcing fibers change by more than a factor of 6, and the maximum deflections change by more than a factor of 5. The use of a more rigid matrix is not necessarily justified from the viewpoint of reducing the deviation of the reflector profile from a specified shape.

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REFERENCES

1. Yu. V. Nemirovskii and B. S. Reznikov, *Strength of Composite Structural Elements* [in Russian], Nauka, Novosibirsk (1986).
2. Ya. M. Grigorenko and A. T. Vasilenko, *Static Problems of Anisotropic Heterogeneous Shells* [in Russian], Nauka, Moscow (1992).
3. S. K. Godunov, "Numerical solution of boundary-value problems for systems of linear ordinary differential equations," *Usp. Mat. Nauk*, **16**, No. 3, 171–174 (1961).
4. S. K. Golushko, V. V. Gorshkov, and A. V. Yurchenko, "Geometrically nonlinear analysis of a reinforced vessel," in: *Dynamics of Continuous Media* (collected scientific papers) [in Russian], No. 114, Novosibirsk (1999), pp. 155–160. ‘
5. S. K. Golushko, V. V. Gorshkov, and A. V. Yurchenko, "Two methods of analysis of conjugate composite structures," in: *Mathematical Modeling of Natural and Man-Caused Catastrophes*, Proc. of 5th Sci. Conf. (Krasnoyarsk, August 17–23, 1999), Krasnoyarsk (1999), pp. 49–55.
6. S. K. Golushko, "Direct and inverse problems in mechanics of composite shells," in: Proc. of the 6th Japan–Russian Joint Symp. Comput. Fluid Dynamics (Nagoya, Sept. 21–23, 1998), Nagoya (1998), pp. 125–130.
7. S. K. Golushko, Yu. V. Nemirovskii, and S. V. Odnoval, "Specific features of behavior of reinforced domes under nonsymmetric loads," *Tr. Novosib. Arkh. Univ.*, No. 1, 37–43 (1998).
8. S. K. Golushko, Yu. V. Nemirovskii, and S. V. Odnoval, "Initial failure of reinforced domes and vaults under nonsymmetric loads," in: *Mathematical Modeling of Natural and Man-Caused Catastrophes*, Proc. 5th Sci. Conf. (Krasnoyarsk, August 17–23, 1999), Krasnoyarsk (1999), pp. 44–48.